Work is **force** times **distance**: $W = Fd$.

- F could be measured in lb and d in ft. Then W is conveniently measured in ft-lb.
- The SI units for F are N (newton), for d they are m (meter), and W is measured in Nm (newtonmeter) or joule. (1 Nm=1 joule)

Example 51. Suppose we wish to lift a 100 lb piano from the ground to the top of a 20 ft building (for instance, by standing on the roof and pulling it up using a rope). The work required for that is

$$\text{work} = (100 \text{ lb})(20 \text{ ft}) = 2000 \text{ ft-lb}.$$

This was easy because the force was constant througout the problem (the piano always weighed 100 lb). It is when the force varies (as in the next example) that we need our calculus skills and mastery of integrals.

Example 52. As before, we wish to lift a 100 lb piano from the ground to the top of a 20 ft building. We are doing so by standing on the roof and pulling it up using a rope. However, this time, we are using a rather heavy rope weighing 0.1 lb/ft and want to take that into account (just pulling up the rope, dangling to the ground, would require some work).

Think about the moment when the piano is x ft off the ground:

- We still need to pull up $20 - x$ ft.
  So, at that moment, the weight (piano plus rope) to be pulled up is
  $$100 + 0.1(20 - x)$$
  pound.
- Hence, to pull up the piano by a tiny amount of $\mathrm{d}x$ feet, the amount of work needed is (roughly) $[100 + 0.1(20 - x)]\mathrm{d}x$ pound.

  [Assuming that $\mathrm{d}x$ is very small, the change in weight is insignificant, so that we can use $W = Fd$.]

To get the total amount of work (in ft-lb), we need to "add" up these small contributions from $x = 0$ to $x = 20$:

$$\text{work} = \int_0^{20} [100 + 0.1(20 - x)]\mathrm{d}x.$$

It only remains to calculate this integral (which is very simple in this case):

$$\text{work} = \int_0^{20} [102 - 0.1x]\mathrm{d}x = \left[102x - \frac{0.1}{2}x^2\right]_0^{20} = 2020 \text{ ft-lb}.$$

Example 53. We started to discuss the case where we want to "lift" a 1600 kg satellite from the ground into orbit, $20,000$ km above the surface.

[These are actually typical values for a GPS satellite.]

Why can we not just multiply the weight times the distance to obtain the work?

Because the gravitational force changes significantly when getting far away from earth.

Ingredients:

- mass and radius of earth
- physical law of attraction: $F = G\frac{m_1 m_2}{d^2}$