Review 40. Consider the same region as above (enclosed by the curves $y=\sqrt{x}$, $y=1$, $x=1$, $x=4$). Now, we revolve this region about the x-axis. What is the volume of the resulting solid?

The solid should look roughly like the coffee mug from Example 37 with a cylindrical hole drilled out.
What do the cross-sections look like now?
Your final answer should be $\frac{9}{2}\pi$.

Review 41. Consider a circle of radius r centered at the origin. Which equation describes the circle?

Answer: The circle consists of all points (x,y) that satisfy $x^2+y^2=r^2$.
This is just the Pythagorean theorem (make a sketch to make sure this is clear to you).

Example 42. We wish to compute the volume of a ball of radius r.

- Which region should we revolve to obtain a ball as our solid of revolution?

  Answer: a half-circle
  A convenient choice is to take the region between $y=\sqrt{r^2-x^2}$ and revolve it about the x-axis.

- Setup the appropriate integral for the volume and evaluate it.

  Answer:

$$\int_{-r}^{r} \pi\left(\sqrt{r^2-x^2}\right)^2 \mathrm{d}x = \int_{-r}^{r} \pi(r^2-x^2)\,\mathrm{d}x = \pi\left[r^2x-\frac{1}{3}x^3\right]_{-r}^{r} = \frac{4}{3}\pi r^3$$

  Sure enough, this is the formula for the volume of a ball that we have seen before (though our memory might be foggy on the exact formula).

Example 43. We had another quick look at the solid obtained by revolving the region enclosed by the curves

$$y=\sqrt{x}, \quad y=0, \quad x=0, \quad x=4,$$

about the x-axis.

- By now, we are experts in computing its volume by "adding" up vertical slices. (Do it!)
- But we can also operate horizontally: namely, consider a small horizontal slice of our region. What does it look like when we revolve only this slice about the x-axis?

  (It should look like a **cylindrical shell**, see Section 6.2 for pictures.)

  We can then compute the volume of our solid by "adding" up the volumes of all these cylindrical shells. Setup the appropriate integral and then find its value.

  (You are encouraged to read through Section 6.2 for assistance.)

  Doing this is a bonus problem!

  See our course website for the details.