Review 36. Derive the formula for the volume of a pyramid of height h whose base is a square with sides of length a.

Solids of revolution

Consider a region (for instance, the region enclosed by a bunch of curves). A solid of revolution is what we obtain when revolving this region about a given line.

(Again, Section 6.1 in the book contains lots of helpful illustrations.)

- Suppose the region is the area between a curve $R(x)$ and the x-axis, between $x = a$ and $x = b$.
- Further, suppose that we revolve this region about the x-axis. Then, slicing vertically, the cross-sections are circles with radius $R(x)$ (so that the cross-sectional area is $A(x) = \pi R(x)^2$).
- Hence, the volume of the resulting solid is

$$\text{vol} = \int_a^b A(x)\mathrm{d}x = \int_a^b \pi R(x)^2\,\mathrm{d}x.$$

Example 37. Consider the region enclosed by the curves $y = \sqrt{x}$, $y = 0$, $x = 1$, $x = 4$ (make a sketch!). If we revolve this region about the x-axis, what is the volume of the resulting solid?

The solid should look roughly like a solid coffee mug (no room for coffee) without handles.

Your final answer should be $\frac{15}{2}\pi$.

Example 38. Consider the region enclosed by the curves $y = \sqrt{x}$, $y = 1$, $x = 1$, $x = 4$ (again, make a sketch). If we revolve this region about the line $y = 1$, what is the volume of the resulting solid?

This solid should look roughly like a bullet.

Your final answer should be $\frac{7}{6}\pi$.

Example 39. Consider the same region as above (enclosed by the curves $y = \sqrt{x}$, $y = 1$, $x = 1$, $x = 4$). Now, we revolve this region about the x-axis. What is the volume of the resulting solid?

The solid should look roughly like the coffee mug from Example 37 with a cylindrical hole drilled out.

What do the cross-sections look like now?

Your final answer should be $\frac{9}{2}\pi$.