Review of the Fundamental Theorem of Calculus

Example 3. Find $\frac{\mathrm{d}}{\mathrm{d}x}\tan(x)$ using $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and the quotient rule.

Example 4. What is $\frac{\mathrm{d}}{\mathrm{d}x}\ln(x)$?

Bonus: Why is that true?

(Recall that $e^{\ln(x)} = x$ and differentiate both sides. What do you conclude?)

Definition 5. If $f(x) = F'(x)$ then we say that $F(x)$ is an **antiderivative** of $f(x)$.

We write: $\int f(x)\mathrm{d}x = F(x) + C$ (This is also called the **indefinite integral** of $f(x)$.)

Example 6. $\int x\,\mathrm{d}x =$

Example 7. $\int x^a\,\mathrm{d}x =$

Example 8. $\int \frac{1-x}{x^3}\,\mathrm{d}x =$

Theorem 9. (Fundamental Theorem of Calculus, part 2)

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a),$$

if $F(x)$ is an antiderivative of $f(x)$.

Recall that $\int_a^b f(x)\,\mathrm{d}x$ denotes the integral of $f(x)$ between a and b. Geometrically, this is the area enclosed between $f(x)$ and the x-axis (assuming that $f(x) \geqslant 0$ to avoid issues of sign).

Example 10. $\int_0^2 x\,\mathrm{d}x =$

First, use the fundamental theorem. Then, make a sketch and give a geometric argument.

Example 11. $\int_1^2 \sqrt{x}\,\mathrm{d}x =$

Theorem 12. (Fundamental Theorem of Calculus)

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(t)\,\mathrm{d}t = f(x)$

(b) $\int_a^x F'(x)\,\mathrm{d}x = F(x) - F(a)$

Note how these two identities reflect that the operations of integration and differentiation are essentially inverses of each other.

Example 13. What is the area under the first hump of the sine function?

Armin Straub
straub@southalabama.edu