Homework #11

MATH 126 — Calculus II
Wednesday, Nov 11

Please print your name:

Problem 1. (9.7.12) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$.

Solution. We apply the ratio test with $a_n = \frac{3^n x^n}{n!}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n}\right| = 3|x| \frac{n!}{(n+1)!} = 3|x| \frac{1}{n+1} \to 0 \text{ as } n \to \infty$$

The ratio test implies that $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$ converges for all x.

The radius of convergence therefore is ∞. □

Problem 2. (9.7.44) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$ and, within this interval, evaluate the series as a function of x.

Solution. This series is obtained from the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ (if $|x| < 1$) by replacing x with $\frac{(x+1)^2}{9}$.

Therefore, $\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n} = \frac{1}{1 - \frac{(x+1)^2}{9}}$ provided that $\left|\frac{(x+1)^2}{9}\right| < 1$ or, equivalently, $|x+1| < 3$.

The condition $|x+1| < 3$ is the same as $x \in (-4, 2)$. The interval of convergence is $(-4, 2)$. □

Armin Straub
straub@southalabama.edu