Finding Lengths of Curves

Find the lengths of the curves in Exercises 1–14. If you have a grapher, you may want to graph these curves to see what they look like.

1. $y = (1/3)(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$
2. $y = x^{3/2}$ from $x = 0$ to $x = 4$
3. $x = (y^3/3) + 1/(4y)$ from $y = 1$ to $y = 3$
4. $x = (y^{3/2}/3) - y^{1/2}$ from $y = 1$ to $y = 9$
5. $x = (y^4/4) + 1/(8y^2)$ from $y = 1$ to $y = 2$
6. $x = (y^3/6) + 1/(2y)$ from $y = 2$ to $y = 3$
7. $y = (3/4)x^{4/3} - (3/8)x^{2/3} + 5, \quad 1 \le x \le 8$
8. $y = (x^3/3) + x^2 + x + 1/(4x + 4), \quad 0 \le x \le 2$
9. $y = \ln x - \dfrac{x^2}{8}$ from $x = 1$ to $x = 2$
10. $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}$ from $x = 1$ to $x = 3$
11. $y = \dfrac{x^3}{3} + \dfrac{1}{4x}, \quad 1 \le x \le 3$
12. $y = \dfrac{x^5}{5} + \dfrac{1}{12x^3}, \quad \dfrac{1}{2} \le x \le 1$
13. $x = \displaystyle\int_0^y \sqrt{\sec^4 t - 1}\, dt, \quad -\pi/4 \le y \le \pi/4$
14. $y = \displaystyle\int_{-2}^x \sqrt{3t^4 - 1}\, dt, \quad -2 \le x \le -1$

T Finding Integrals for Lengths of Curves

In Exercises 15–22, do the following.

a. Set up an integral for the length of the curve.

b. Graph the curve to see what it looks like.

c. Use your grapher's or computer's integral evaluator to find the curve's length numerically.

15. $y = x^2, \quad -1 \le x \le 2$
16. $y = \tan x, \quad -\pi/3 \le x \le 0$
17. $x = \sin y, \quad 0 \le y \le \pi$
18. $x = \sqrt{1 - y^2}, \quad -1/2 \le y \le 1/2$
19. $y^2 + 2y = 2x + 1$ from $(-1, -1)$ to $(7, 3)$
20. $y = \sin x - x \cos x, \quad 0 \le x \le \pi$
21. $y = \displaystyle\int_0^x \tan t\, dt, \quad 0 \le x \le \pi/6$
22. $x = \displaystyle\int_0^y \sqrt{\sec^2 t - 1}\, dt, \quad -\pi/3 \le y \le \pi/4$

Theory and Examples

23. **a.** Find a curve through the point (1, 1) whose length integral (Equation 3) is

$$L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx.$$

b. How many such curves are there? Give reasons for your answer.