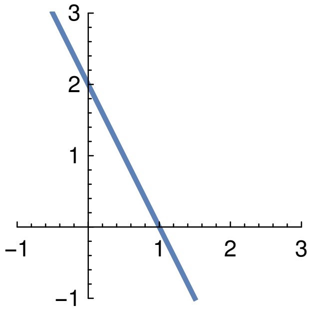
Check out the dancing ghosts again!

Cute as they are...a few of them need to seriously work on their moves:

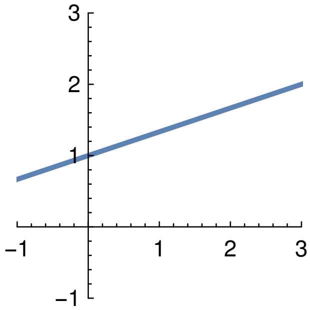
- -2^x dances as if he was 2^{-x}
- $-\sqrt{x}$ dances as if he was $\sqrt{-x}$
- $x = 0$ dances as if he was $y = 0$

Estimate the slopes! Equations?



$\text{slope} = -2$

$y = -2x + 2$



$\text{slope} = \frac{1}{3}$

$y = \frac{1}{3}x + 1$

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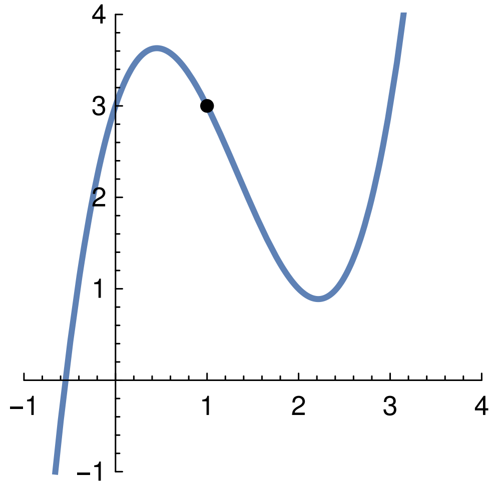
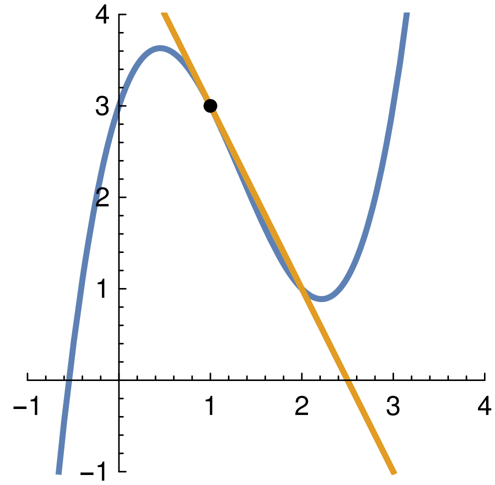
1 The derivative

At, say, $x = \boxed{1}$ there is a **tangent line** approximating $f(x)$.

We write $f'(\boxed{1})$ for the slope of this tangent line.



$f(1) = 3$



$f'(1) = -2$

$f'(x)$ is called the **derivative** of $f(x)$.
Another common notation: $\frac{\mathrm{d}}{\mathrm{d}x} f(x) = f'(x)$

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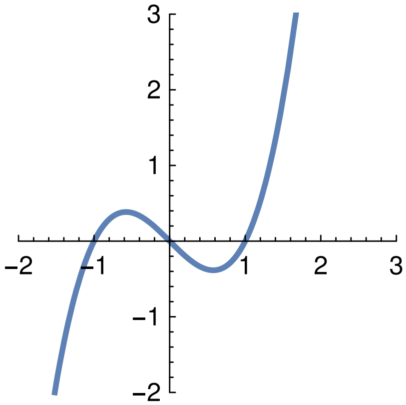
Example 1. For $f(x)$ as in the graph, estimate the following:

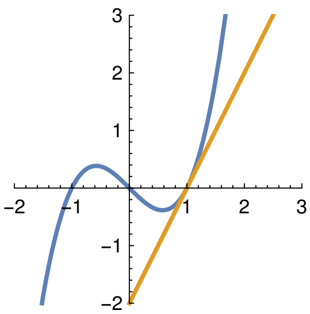


(a) $f'(1)$

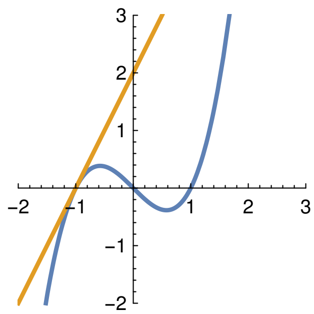
(b) $f'(-1)$

(c) $f'(0)$

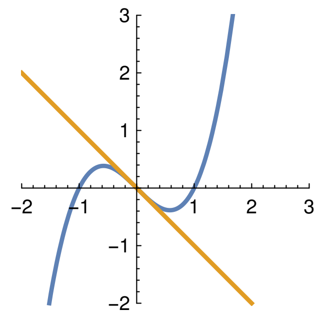
(d) $f'(\frac{1}{2})$



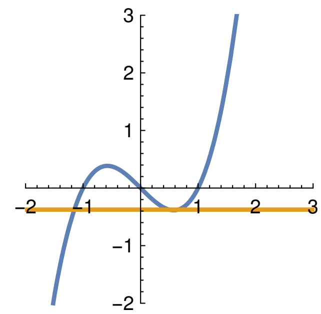
$f'(1) = 2$



$f'(-1) = 2$



$f'(0) = -1$



$f'(\frac{1}{2}) = 0$

Example 2. Find an equation for the tangent line at $x = 1$.

Solution. Line has slope $f'(1) = 2$ and goes through $(\boxed{1}, \boxed{0})$.

Hence, an equation is $y - \boxed{0} = 2(x - \boxed{1})$.

[Or, if preferred, $y = 2x - 2$, the slope-intercept form.]

1.1 Computing derivatives—a trivial case

(obvious) If $f(x) = mx + b$, then $f'(x) = m$.

Why? This is a line. At every point, it has slope m.

2 The power rule

(power rule) If $f(x) = x^r$, then $f'(x) = r x^{r-1}$.

Example 3. What is $f'(x)$ in each case?

(a) $f(x) = x^2$ $\quad$ $f'(x) = 2x^1 = 2x$

(b) $f(x) = x^4$ $\quad$ $f'(x) = 4x^3$

(c) $f(x) = 2^6$ $\quad$ $f'(x) = 0$ (this is a horizontal line: $f(x) = 64$)

(d) $f(x) = \sqrt{x} = x^{1/2}$ $\quad$ $f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(e) $f(x) = \frac{1}{x^2} = x^{-2}$ $\quad$ $f'(x) = -2x^{-3} = -\frac{2}{x^3}$

Example 4. Find an equation for the tangent line to the graph of $f(x) = \frac{1}{x^2}$ at the point $\left(3, \frac{1}{9}\right)$.

Solution. Since $f'(x) = -\frac{2}{x^3}$, the slope is $f'(3) = -\frac{2}{3^3} = -\frac{2}{27}$.

Line goes through $\left(\boxed{3}, \boxed{\frac{1}{9}}\right)$.

Hence, an equation is $y - \boxed{\frac{1}{9}} = -\frac{2}{27}\left(x - \boxed{3}\right)$.

Optionally, in slope-intercept form: $y = -\frac{2}{27}x + \frac{1}{3}$
[MyLabsPlus should accept any form.]

Play time! Plot $f(x)$ and tangent line using GeoGebra.

https://www.geogebra.org/graphing

Does the tangent line indeed touch the graph of $f(x)$ at the point $\left(3, \frac{1}{9}\right)$?

Homework. Determine the tangent line at $x = -1$.

Again, plot both $f(x)$ and the tangent line in GeoGebra.
[The final answer in slope-intercept form is $y = 2x + 3$.]

3 Basic rules for differentiation

(constant rule) $\frac{\mathrm{d}}{\mathrm{d}x}[k f(x)] = k f'(x)$ if k is a constant

(sum rule) $\frac{\mathrm{d}}{\mathrm{d}x}[f(x) + g(x)] = f'(x) + g'(x)$

Example 5. Let $f(x) = -2x^4$. What is $f'(x)$?

Solution. The derivative of x^4 is $4x^3$.

By the constant rule, $f'(x) = -2 \cdot 4x^3 = -8x^3$.

Example 6. Let $f(x) = -2x^4 + 3x^5$. What is $f'(x)$?

Solution. The derivative of $-2x^4$ is $-8x^3$.

The derivative of $3x^5$ is $15x^4$.

By the sum rule, $f'(x) = -8x^3 + 15x^4$.

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