The next online test (Chapter 3/4) is open now:

You have until **Wednesday, Nov 1**, to complete it on MyLabsPlus.

Until the test, you can still submit all Chapter 3/4 assignments and quizzes (even if you missed the due date).

(exponentials/logarithms)

$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x \qquad \frac{\mathrm{d}}{\mathrm{d}x} \ln(x) = \frac{1}{x}$

$e^{x \ln(5)} = 5^x$

$\frac{\mathrm{d}}{\mathrm{d}x} e^{5x} = 5e^{5x}$

https://www.amazon.com/A-wild-EXPONENTIAL-FUNCTION-appeared/dp/B01MZIIRWZ

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1 Online test on Chapters 3 and 4

- You have until Wednesday, Nov 1, to complete it on MyLabsPlus.
- The password is "euler".
- You have 60 min for 10 questions.
  - 7 of these questions ask for a derivative.

    For instance, differentiate the following:
    - $x^{10}(2x^4-1)^{18}$
    - $\frac{2x^2+x-1}{3x^2+5}$
    - $f(g(x))$ with $f(x)=\sqrt{x}$, $g(x)=2x^3-6$
    - x^8e^x
    - $\sqrt{e^x+19}$
    - $5e^{x/7}$
    - $\ln(2x^3-5\sqrt{x}+2)$
  - 2 questions about working with exponentials
    - review Examples 1, 2, 3 from Slides #8
  - one question to find local min/max
    - review the two problems on 4.3 homework
    - as well as the next example

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Example 1. Find the local extrema of $f(x) = (6 + x)e^{-4x}$. Classify them as min/max.

Solution.

- To find the critical points, we solve $f'(x) = 0$.

$$\begin{aligned} f'(x) &= 1 \cdot e^{-4x} + (6 + x) \cdot \left(\tfrac{\mathrm{d}}{\mathrm{d}x} e^{-4x}\right) && \text{(product rule)} \\ &= e^{-4x} + (6 + x) \cdot (-4e^{-4x}) && \text{(chain rule)} \\ &= (-23 - 4x)e^{-4x} \end{aligned}$$

  $(-23 - 4x)e^{-4x} = 0$

  $-23 - 4x = 0$

  Hence, $x = -\frac{23}{4}$ is the only critical point.

- We need to decide if there is a min/max at $x = -\frac{23}{4}$

  $\Longrightarrow$ first-derivative test or second-derivative test

  The first-derivative test is a bit easier here (but trickier?):

  The sign of $f'(x)$ is the same as the sign of $-23 - 4x$.

  The latter is a line, sloped downwards.

  $\Longrightarrow$ $f'(x)$ changes from $+$ to $-$ at $x = -\frac{23}{4}$.

  $\Longrightarrow$ $f(x)$ has a max at $x = -\frac{23}{4}$.

Example 2. Carry out the second-derivative test.

Solution. $f'(x) = (-23 - 4x)e^{-4x}$

$$\begin{aligned} f''(x) &= -4 \cdot e^{-4x} + (-23 - 4x) \cdot \left(\tfrac{\mathrm{d}}{\mathrm{d}x} e^{-4x}\right) && \text{(product rule)} \\ &= -4e^{-4x} + (-23 - 4x) \cdot (-4e^{-4x}) && \text{(chain rule)} \\ &= (88 + 16x)e^{-4x} \end{aligned}$$

In particular, $f''\left(-\frac{23}{4}\right) = (88 - 4 \cdot 23)e^{23} = -4e^{23} < 0$.

Hence, $f(x)$ has a max at (the critical point) $x = -\frac{23}{4}$.

2 Legend of the grains on the chessboard

1

2

4

8

$\vdots$

2^{63} on square 64

Maren Winter / shutterstock.com

The total number of grains is (surprisingly?) large:

$18,446,744,073,709,551,615 = 2^{64} - 1$

If you can count 10 grains per second, you'd be counting about

$58,494,241,736$ years!

Phycisists estimate our universe to be $13,800,000,000$ years old.

https://en.wikipedia.org/wiki/Wheat_and_chessboard_problem

http://www.npr.org/sections/krulwich/2012/09/15/160879929/that-old-rice-grains-on-the-chessboard-con-with-a-new-twist

Example 3. Why is $\frac{\mathrm{d}}{\mathrm{d}x}\ln(x) = \frac{1}{x}$?

Solution. Recall that $e^{\ln(x)} = x$.

We now differentiate both sides:

(use chain rule with $f(x) = e^x$, $g(x) = \ln(x)$ on the left)

$e^{\ln(x)} \cdot \ln'(x) = 1$

Hence, $\ln'(x) = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$.

Example 4. $\frac{\mathrm{d}}{\mathrm{d}x}2^x =$

Solution. Recall that $2^x = e^{x\ln(2)}$.

$\frac{\mathrm{d}}{\mathrm{d}x}2^x = \frac{\mathrm{d}}{\mathrm{d}x}e^{x\ln(2)} = e^{x\ln(2)} \cdot \ln(2) = \ln(2) \cdot 2^x$

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