

1 Online tests

MyLabsPlus usually does not give partial credit.

Let me know if you really deserve partial credit on a problem (for instance, two answers on optimization problem swapped).

To avoid losing points, make sure to revisit relevant homeworks/quizzes beforehand; guided by list of questions to expect.

Good news!

If beneficial, the score on your final exam replaces the lowest test score.

The next online test (Chapter 3/4) is 10/28 to 11/1/2017.

2 Review exponentials/logs over Fall break

- $a^x \cdot a^y = a^{x+y}$
- $\frac{1}{a^x} = a^{-x}$
- $a^x \cdot b^x = (ab)^x$
- $\log_a(a^x) = x$ and $a^{\log_a(r)} = r$
- $\log_a(rs) = \log_a(r) + \log_a(s)$
- $\log_a\left(\frac{1}{r}\right) = -\log_a(r)$

(chain rule)

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

(product rule)

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

(quotient rule)

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

3 Chain rule

Example 1. (warmup) If $f(x) = x + \sqrt{x+1}$ and $g(x) = x^4 + 1$,
then $f(g(x)) = g(x) + \sqrt{g(x)+1} = x^4 + 1 + \sqrt{x^4 + 2}$.

$$\text{(generalized power rule)} \quad \frac{d}{dx}[g(x)^r] = r g(x)^{r-1} \cdot g'(x)$$

The generalized power rule follows from the chain rule with $f(x) = x^r$.

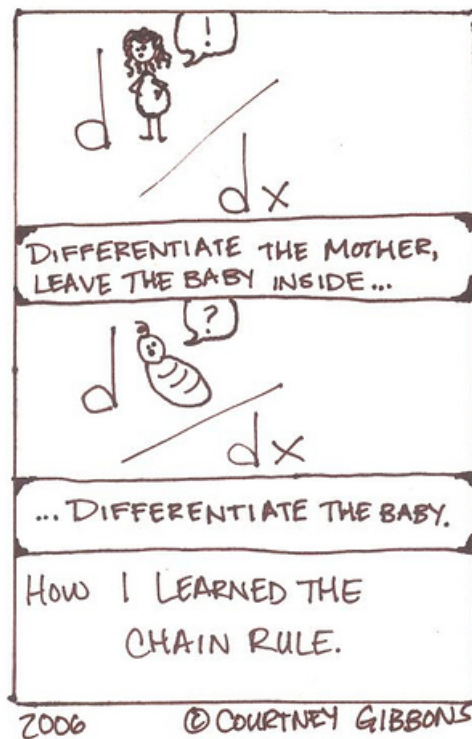
(chain rule)

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Why?

In short form: $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$

(Here, $z = g(x)$ and $y = f(z) = f(g(x))$.)



Example 2.

(a) Write $h(x) = (x^3 + 7)^{10}$ in the form $f(g(x))$.

(b) Differentiate $h(x) = (x^3 + 7)^{10}$.

Solution.

(a) The natural choice is: $f(x) = x^{10}$ and $g(x) = x^3 + 7$

(b) First, we compute $f'(x) = 10x^9$ and $g'(x) = 3x^2$.

$$\begin{aligned}h'(x) &= f'(g(x)) \cdot g'(x) \\ &= 10g(x)^9 \cdot (3x^2) \\ &= 10(x^3 + 7)^9 \cdot (3x^2) \\ &= 30x^2(x^3 + 7)^9\end{aligned}$$

Example 3.

(a) Write $h(x) = \sqrt{x^2 - 3\sqrt{x}}$ in the form $f(g(x))$.

(b) Differentiate $h(x) = \sqrt{x^2 - 3\sqrt{x}}$.

Solution.

(a) The natural choice is: $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3\sqrt{x}$

(b) First, we compute $f'(x) = \frac{1}{2}x^{-1/2}$ and $g'(x) = 2x - \frac{3}{2}x^{-1/2}$.

$$\begin{aligned}h'(x) &= f'(g(x)) \cdot g'(x) \\ &= \frac{1}{2}g(x)^{-1/2} \cdot \left(2x - \frac{3}{2}x^{-1/2}\right) \\ &= \frac{1}{2}(x^2 - 3\sqrt{x})^{-1/2} \cdot \left(2x - \frac{3}{2}x^{-1/2}\right)\end{aligned}$$

4 Product and quotient rule

$$\text{(product rule)} \quad \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Example 4. Differentiate $h(x) = (x^2 + 3)(2x^4 - 1)$.

Solution. (by multiplying out)

$$h(x) = 2x^6 + 6x^4 - x^2 - 3$$

$$h'(x) = 12x^5 + 24x^3 - 2x$$

Solution. (via product rule)

Write $h(x) = f(x)g(x)$ with $f(x) = x^2 + 3$ and $g(x) = 2x^4 - 1$.

$$\begin{aligned} h'(x) &= f'(x)g(x) + f(x)g'(x) \\ &= (2x)(2x^4 - 1) + (x^2 + 3)(8x^3) \\ &= (4x^5 - 2x) + (8x^5 + 24x^3) \\ &= 12x^5 + 24x^3 - 2x \end{aligned}$$

Example 5. Differentiate $h(x) = x^2(x^3 + 7)^{10}$.

(Multiplying out is still possible, but would be a huge pain.)

Solution.

Write $h(x) = f(x)g(x)$ with $f(x) = x^2$ and $g(x) = (x^3 + 7)^{10}$.

Clearly, $f'(x) = 2x$. We computed $g'(x)$ two examples ago:

$$g'(x) = 10(x^3 + 7)^9 \cdot 3x^2 = 30x^2(x^3 + 7)^9 \quad \text{(chain rule)}$$

$$\begin{aligned}
h'(x) &= f'(x)g(x) + f(x)g'(x) \\
&= 2x(x^3 + 7)^{10} + x^2 \cdot 30x^2(x^3 + 7)^9 \\
&= 2x(x^3 + 7)^{10} + 30x^4(x^3 + 7)^9 \quad (\text{fine final answer}) \\
&= (2x(x^3 + 7) + 30x^4)(x^3 + 7)^9 \\
&= (32x^4 + 14x)(x^3 + 7)^9
\end{aligned}$$

(quotient rule) $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Example 6. $h(x) = \frac{1}{x}$ two ways

Solution. (power rule)

Since $h(x) = x^{-1}$, $h'(x) = -x^{-2} = -\frac{1}{x^2}$.

Solution. (quotient rule)

Write $h(x) = \frac{f(x)}{g(x)}$ with $f(x) = 1$ and $g(x) = x$.

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} = \frac{0 \cdot x - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$$

Example 7. Differentiate $h(x) = \frac{x^2 - 2}{3x + 7}$.

Solution.

Write $h(x) = \frac{f(x)}{g(x)}$ with $f(x) = x^2 - 2$ and $g(x) = 3x + 7$.

$$\begin{aligned}
h'(x) &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\
&= \frac{(2x) \cdot (3x + 7) - (x^2 - 2) \cdot 3}{(3x + 7)^2} \\
&= \frac{(6x^2 + 14x) - (3x^2 - 6)}{(3x + 7)^2} \\
&= \frac{3x^2 + 14x + 6}{(3x + 7)^2}
\end{aligned}$$

Assignments.

- check out Sections 3.1, 3.2 in the book
- do “3.1. Product and Quotient rule” (6 questions)
- do “3.2. Chain rule” (6 questions)
- take “chapter 3 quiz” (5 questions)
All five questions are taken from homework assignments.
- review basic algebra of exponentials/logs

5 Timeline until next test

The next online test (Chapter 3/4) is 10/28 to 11/1/2017.

After this class:

- do “3.1. Product and Quotient rule” (6 questions)
- do “3.2. Chain rule” (6 questions)
- take “chapter 3 quiz” (5 questions)
- review basic algebra of exponentials/logs

After next class (Oct 19/20):

- do “4.1, 4.2. Exponential functions and e^x ” (10 questions)
- do “4.3 On the derivative of e^x .” (6 questions)
- do “4.4, 4.5, 4.6. About $\ln(x)$ ” (9 questions)
- take “chapter 4 quiz” (7 questions)

Class thereafter (Oct 26/27) used for exam prep.

- take “Test on chapters 3 and 4” (10 questions, 60min)