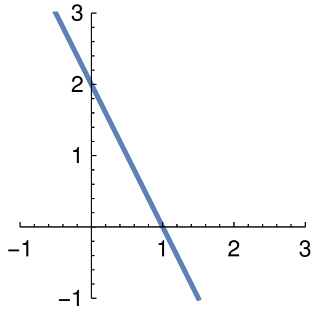
Check out the dancing ghosts again!

Cute as they are...a few of them need to seriously work on their moves:

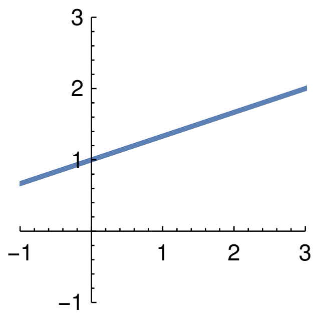
- -2^x dances as if he was 2^{-x}
- $-\sqrt{x}$ dances as if he was $\sqrt{-x}$
- $x = 0$ dances as if he was $y = 0$

Estimate the slopes! Equations?



slope $= -2$

$y = -2x + 2$



slope $= \frac{1}{3}$

$y = \frac{1}{3}x + 1$

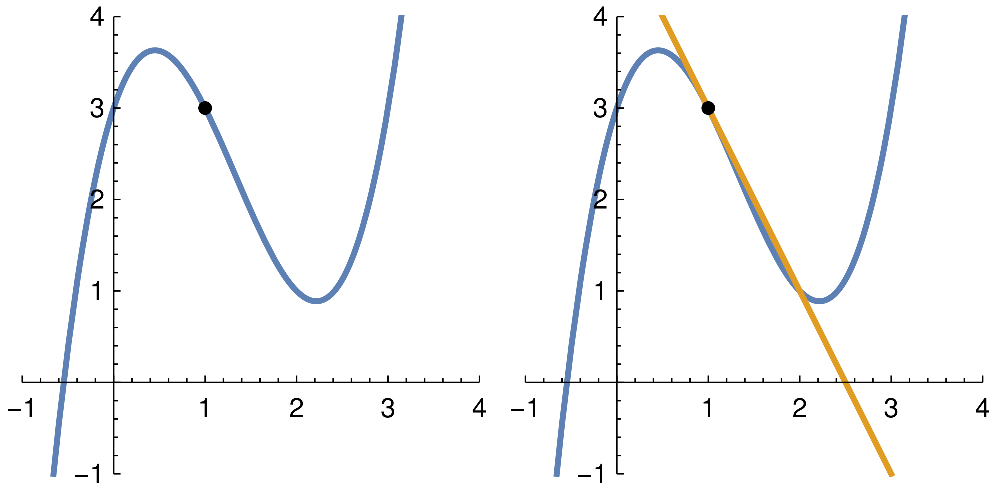
by chibipandora @ deviantART

Armin Straub
straub@southalabama.edu

1 The derivative

At, say, $x = \boxed{1}$ there is a **tangent line** approximating $f(x)$.

We write $f'(\boxed{1})$ for the slope of this tangent line.

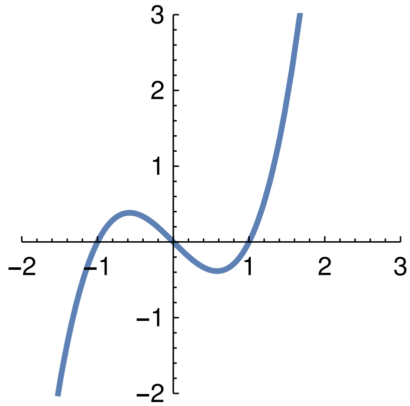


$f(1) = 3$ $\qquad$ $f'(1) = -2$

> $f'(x)$ is called the **derivative** of $f(x)$.
> Another common notation: $\frac{\mathrm{d}}{\mathrm{d}x} f(x) = f'(x)$

Armin Straub
straub@southalabama.edu

Example 1. For $f(x)$ as in the graph, estimate the following:

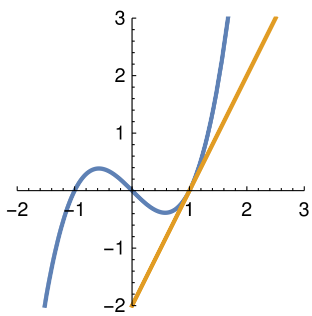


(a) $f'(1)$

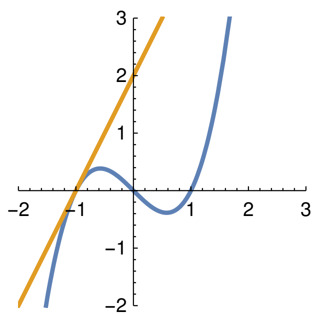
(b) $f'(-1)$

(c) $f'(0)$

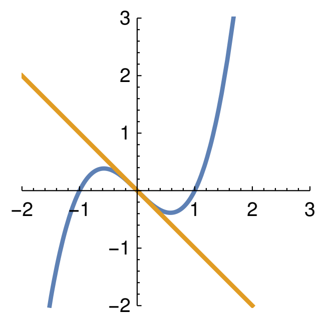
(d) $f'(\frac{1}{2})$



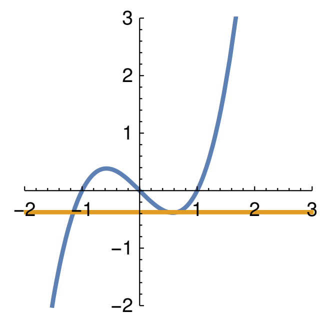
$f'(1) = 2$



$f'(-1) = 2$



$f'(0) = -1$



$f'(\frac{1}{2}) = 0$

Example 2. Find an equation for the tangent line at $x = 1$.

Solution. Line has slope $f'(1) = 2$ and goes through $(\boxed{1}, \boxed{0})$.

Hence, an equation is $y - \boxed{0} = 2(x - \boxed{1})$.

[Optionally, this simplies to $y = 2x - 2$, the slope-intercept form.]

1.1 Computing derivatives—a trivial case

(obvious) If $f(x) = mx + b$, then $f'(x) = m$.

Why? This is a line. At every point, it has slope m.

Example 3. If $f(x) = -\pi^3$, then $f'(x) = 0$.

In that case, $f(x)$ is a horizontal line (i.e. with slope 0).

[Just to make sure: $-\pi^3 \approx -31.006$ is a constant.]

Armin Straub
straub@southalabama.edu

2 The power rule

(power rule) If $f(x) = x^r$, then $f'(x) = r x^{r-1}$.

Example 4. What is $f'(x)$ in each case?

(a) $f(x) = x^2$ $\quad$ $f'(x) = 2x^1 = 2x$

(b) $f(x) = x^4$ $\quad$ $f'(x) = 4x^3$

(c) $f(x) = 2^6$ $\quad$ $f'(x) = 0$ (see Example 3; here, $f(x) = 64$)

(d) $f(x) = \sqrt{x} = x^{1/2}$ $\quad$ $f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(e) $f(x) = \frac{1}{x^2} = x^{-2}$ $\quad$ $f'(x) = -2x^{-3} = -\frac{2}{x^3}$

Example 5. Find an equation for the tangent line to the graph of $f(x) = \frac{1}{x^2}$ at the point $\left(3, \frac{1}{9}\right)$.

Solution. Since $f'(x) = -\frac{2}{x^3}$, the slope is $f'(3) = -\frac{2}{3^3} = -\frac{2}{27}$.

Line goes through $\left(\boxed{3}, \boxed{\frac{1}{9}}\right)$.

Hence, an equation is $y - \boxed{\frac{1}{9}} = -\frac{2}{27}(x - \boxed{3})$.

Optionally, this simplifies to $y = -\frac{2}{27}x + \frac{1}{3}$ (slope-intercept form).

[MyLabsPlus should accept any form.]

Play time! Plot $f(x)$ and tangent line using GeoGebra.

https://www.geogebra.org/graphing

Does the tangent line indeed touch the graph of $f(x)$ at the point $\left(3, \frac{1}{9}\right)$?

Homework. Determine the tangent line at $x = -1$.

Again, plot both $f(x)$ and the tangent line in GeoGebra.

(The final answer in slope-intercept form is $y = 2x + 3$.)

Armin Straub
straub@southalabama.edu

3 Some rules for differentiation

Go through Section 1.6 in the book.

Example 6. Let $f(x) = -2x^4$. What is $f'(x)$?

Solution. The derivative of x^4 is $4x^3$.
For us, y is scaled by -2 (a constant!). The slopes are scaled likewise.
Hence, $f'(x) = -2 \cdot 4x^3 = -8x^3$.

Example 7. Let $f(x) = -2x^4 + 3x^5$. What is $f'(x)$?

Solution. The derivative of $-2x^4$ is $-8x^3$.
The derivative of $3x^5$ is $15x^4$.
Hence, $f'(x) = -8x^3 + 15x^4$.

Play time! Do the previous example in GeoGebra using:

`f(x)=-2x^4+3x^5` (then press ENTER)

`f'(x)`

https://www.geogebra.org/graphing

(generalized power rule)
If $f(x) = g(x)^r$, then $f'(x) = r\,g(x)^{r-1} \cdot g'(x)$.

Example 8. If $f(x) = (3x^2 - 1)^2$, what is $f'(x)$?

Solution. (by expanding)
$f(x) = 9x^4 - 6x^2 + 1$, so that $f'(x) = 36x^3 - 12x$.

Solution. (using generalized power rule)
$f(x) = g(x)^2$ with $g(x) = 3x^2 - 1$. First, $g'(x) = 6x$.
Hence, $f'(x) = 2g(x) \cdot g'(x) = 2(3x^2 - 1) \cdot 6x$.
[Optionally, we expand $2(3x^2 - 1) \cdot 6x = 36x^3 - 12x$, as before.]

Armin Straub
straub@southalabama.edu

Example 9. What is $f'(x)$ in each case?

(a) $f(x) = (2x+3)^{10}$ [Here, $g(x) = 2x+3$.]

$f'(x) = 10(2x+3)^9 \cdot 2 = 20(2x+3)^9$

(b) $f(x) = \frac{1}{x^2+1} = (x^2+1)^{-1}$ [Here, $g(x) = x^2+1$.]

$f'(x) = -(x^2+1)^{-2} \cdot 2x = -\frac{2x}{(x^2+1)^2}$

4 Higher derivatives

The derivative of the derivative is the **second derivative**.

It is denoted $f''(x)$ or $\frac{\mathrm{d}^2}{\mathrm{d}x^2} f(x)$. Or, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

Similarly, but less important, there is a third derivative and so on...

Example 10. Let $y = -2x^4 + 3x$. Find the first and second derivatives.

(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = -8x^3 + 3$

(b) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -24x^2$

Example 11. Determine: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}(2x^3 - x + 1)\Big|_{x=5}$

This is the same as setting $f(x) = 2x^3 - x + 1$ and asking for $f''(5)$.

Solution.

$\frac{\mathrm{d}}{\mathrm{d}x}(2x^3 - x + 1) = 6x^2 - 1$

$\frac{\mathrm{d}^2}{\mathrm{d}x^2}(2x^3 - x + 1) = 12x$

$\frac{\mathrm{d}^2}{\mathrm{d}x^2}(2x^3 - x + 1)\Big|_{x=5} = 12 \cdot 5 = 60$

Homework. Do this computation in GeoGebra.

First, write `f(x)=2x^3-x+1`, then `f''(5)`.

Assignments.

- do "1.3. The derivative" (10 questions)
- check out Section 1.3 in the book

  (Book available in MyLabsPlus under "eText".)
- take the quiz "1.2, 1.3. Power rule quiz" (4 questions)
- check out Section 1.6 in the book
- do "1.6. Rules for derivatives" (6 questions)
- begin "1.7. More on derivatives" (first 6 of 8 questions)

5 Taking your first quiz

Your first quiz consists of 4 questions, much like the following:

- If $f(x) = x^5$, then $f'(x) = \boxed{}$.
- If $f(x) = 2^6$, then $f'(x) = \boxed{}$.
- $\frac{\mathrm{d}}{\mathrm{d}x}(x^{11}) = \boxed{}$.
- Find the slope of the tangent line to the graph of $y = x^3$ at the point where $x = -2/7$. The slope is $\boxed{}$.

Final answers. $5x^4$, 0, $11x^{10}$, $\frac{12}{49}$