- $\frac{d}{dx}\sin(x) = \cos(x)$ **(derivative of $\sin(x)$)**
- $\frac{d}{dx}\cos(x) = -\sin(x)$ **(derivative of $\sin(x)$)**

A familiar limit. Note that $\left[\frac{d}{dx}\sin(x)\right]_{x=0} = \lim_{h\to 0}\frac{\sin(0+h)-\sin(0)}{h} = \lim_{h\to 0}\frac{\sin(h)}{h} = 1 = \cos(0)$.

Derivatives of the other trig functions can be found from these and the product/quotient rule.

Example 60. (derivatives of other trig functions)

(a) $\frac{d}{dx}\tan(x)$

(b) $\frac{d}{dx}\sec(x)$

(c) $\frac{d}{dx}\cot(x)$

Solution.

(a) $\frac{d}{dx}\tan(x) = \frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\cos(x)\cos(x) - (-\sin(x))\sin(x)}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$

(b) $\frac{d}{dx}\sec(x) = \frac{d}{dx}\frac{1}{\cos(x)} = \frac{0\cdot\cos(x) - 1\cdot(-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)}\frac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$

(c) $\frac{d}{dx}\cot(x) = \frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)} = -\frac{1}{\sin^2(x)} = -\csc^2(x)$

Example 61.

(a) $\frac{d}{dx}[2x^3 - 5\cos(x)]$

(b) $\frac{d}{dx}[2x^3\sin(x)]$

(c) $\frac{d^{22}}{dx^{22}}\sin(x)$

Solution.

(a) $\frac{d}{dx}[2x^3 - 5\cos(x)] = 6x^2 + 5\sin(x)$

(b) $\frac{d}{dx}[2x^3\sin(x)] = 6x^2\sin(x) + 2x^3\cos(x)$

(c) $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d^2}{dx^2}\sin(x) = -\sin(x)$, $\frac{d^3}{dx^3}\sin(x) = -\cos(x)$, $\frac{d^4}{dx^4}\sin(x) = \sin(x)$.
In other words, after four derivatives, we are back to the original function!
Hence, $\frac{d^{20}}{dx^{20}}\sin(x) = \sin(x)$ and so $\frac{d^{22}}{dx^{22}}\sin(x) = \frac{d^2}{dx^2}\sin(x) = -\sin(x)$.

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