In order to improve on the bound \( a \leq 0 \) in [Str08, Corollary 2] we prove that \( h_{a,0} \) is positive only if \( a \leq 3/4 \). The following proof has been kindly suggested by Bruno Salvy. Thank you!

**Lemma 1.** The function \( h_{a,0} \) defined by

\[
h_{a,0}(x, y, z) = \frac{1}{1 - (x + y + z) + a(xy + yz + zx)}
\]

is positive only if \( a \leq 3/4 \).

**Proof.** Now, suppose \( a > 3/4 \), and write \( a = \frac{3}{4}(1 + \frac{t^2}{2}) \) for \( t > 0 \). If \( h_{a,0} \) is positive, then so is

\[
H_a(x) := h_{a,0}\left(\frac{2}{3}x, \frac{2}{3}x, \frac{2}{3}x\right) = \frac{1}{1 - 2x + (1 + t^2)x^2}.
\]

Observe that

\[
t x H_a(x) = \sum_{n \geq 0} \text{im}((1 + it)^n)x^n = \sum_{n \geq 0} (1 + t^2)^{n/2} \sin(n \arctan(t))x^n.
\]

We thus see that the Taylor coefficients of \( H_a \) change sign infinitely often. In order for \( h_{a,0} \) to be positive we therefore need \( a \leq 3/4 \). \( \square \)

**Corollary 2.** Let \( a \leq 3/4 \). Then \( h_{a,b} \) is positive only if \( b \leq 2 - 3a + 2(1 - a)^{3/2} \).

**Bibliography**